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MATDIP301

**Third Semester B.E Degree Examination, June/July 2017**  
**Advanced Mathematics – I**

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions.**

- 1 a. Express :  $\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}$  in the form of  $a + ib$ . (07 Marks)
- b. Find the modulus and amplitude of the complex number  $1 - \cos \alpha + i \sin \alpha$ . (06 Marks)
- c. Express the complex number  $\sqrt{3} + i$  in the polar form. (07 Marks)
- 2 a. Find the  $n^{\text{th}}$  derivative of  $\log(ax + b)$ . (07 Marks)
- b. Find the  $n^{\text{th}}$  derivative of  $\frac{x}{(x-1)(2x+3)}$ . (06 Marks)
- c. If  $y = \sin^{-1} x$ , prove that :  $(1-x^2)y_{n+2} - (2n+1)x y_{n+1} - n^2 y_n = 0$ . (07 Marks)
- 3 a. Using Taylor's theorem, expand  $\sin x$  in power of  $(x - \pi/2)$ . (07 Marks)
- b. Obtain the Maclaurin's series expansion of the function  $\sqrt{1 + \sin 2x}$  up to the term containing  $x^4$ . (06 Marks)
- c. State and prove Euler's theorem. (07 Marks)
- 4 a. Find the total derivative of  $z = xy^2 + x^2y$  where  $x = at$ ,  $y = 2at$ , and also verify the result by direct substitution. (07 Marks)
- b. If  $u = f(y - z, z - x, x - y)$  prove that :  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ . (06 Marks)
- c. if  $x = u(1 - v)$  and  $y = uv$ , find  $J = \frac{\partial(x, y)}{\partial(u, v)}$  and  $J' = \frac{\partial(u, v)}{\partial(x, y)}$  and also verify  $J \cdot J' = 1$ . (07 Marks)
- 5 a. Obtain the reduction formula for  $\int \cos^n x \cdot dx$ . (07 Marks)
- b. Evaluate :  $\int_0^2 \frac{x^4}{\sqrt{4-x^2}} \cdot dx$ . (06 Marks)
- c. Evaluate :  $\int_1^2 \int_1^3 xy^2 dx dy$ . (07 Marks)
- 6 a. Evaluate :  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} x y z dz dy dx$ . (07 Marks)
- b. Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ . (06 Marks)
- c. Prove that  $\beta(m, n) = \frac{\Gamma_m \Gamma_n}{\Gamma(m+n)}$ . (07 Marks)

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- 7 a. Solve  $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ . (07 Marks)
- b. Solve  $x^2y dx - (x^3 + y^3) dy = 0$ . (06 Marks)
- c. Solve  $\frac{dy}{dx} + y \cot x = \cos x$ . (07 Marks)
- 8 a. Solve :  $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 0$ . (05 Marks)
- b. Solve  $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 9y = 3e^{-4x}$ . (05 Marks)
- c. Solve :  $y'' + 2y' + y = e^{-x} + \cos 2x$ . (05 Marks)
- d. Solve :  $\frac{d^2y}{dx^2} - 4y = x \sin 2x$ . (05 Marks)

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